Lecture 32 - Transverse and longitudinal Waves

Waves on a wire - Guitar strings

Waves travelling down a wire or a rope are transverse waves, and their wavespeed increases the greater the tension in the wire and reduces as the mass of the string or wire increases. The precise formula is,

$$v = (\frac{F}{\mu})^{1/2} \tag{1}$$

Travelling waves are like water waves that move in a particular direction. A different sort of wave occurs on guitar string because both ends of the string are fixed. The vibrations of the string do not seem to move with the wavespeed v. Why not?? These waves seem to stay in one place and for this reason are called standing waves. They can be thought of as a sum of travelling waves, where each of the travelling waves in the string moves with the wavespeed v. For standing waves to occur there needs to be special relationships between the wavelength of the wave and the length of the string. In the case of a guitar, the wave amplitude must be zero at each end of the string, so that an integral number of half wavelengths must "fit" in the string length L. We must then have,

$$\frac{1}{2}\lambda_n n = L \quad \text{or} \quad \lambda_n = \frac{2L}{n} \tag{2}$$

The vibrations of the string cause a pressure wave in the air and this pressure wave is the sound which we hear, as we will return to later. Notice that waves on a guitar string are transverse standing waves. The fundamental wavelength is $\lambda_1 = 2L$. The frequency associated with this fundamental $f_1 = v/\lambda_1$. The frequencies associated with the other wavelengths are given by,

$$f_n = \frac{v}{\lambda_n} = \frac{n}{2L}v = nf_1 \tag{3}$$

Sound Waves

Applications

Sound waves are longitudinal waves and have different names depending on the frequency of the waves. (Human) Audible frequencies lie in the range $20Hz < f < 20,000Hz$. Infrasonic waves lie below the audible frequency range, while ultrasonic waves have frequencies above the audible. Ultrasonic

waves are used extensively in biology ranging from low intensity applications in imaging (e.g. ultrasound) to high intensity applications in surgery (e.g. CUSA). A piezoelectric device can be used to generate sound waves as the piezo material oscillates at the same frequency as an applied oscillatory voltage. The percentage of the sound wave that is reflected is given by,

$$PR = (\frac{\rho_h - \rho_l}{\rho_h + \rho_l})^2 * 100 \tag{4}$$

Contrast is stongest when the differences in density are largest.
The speed of sound
    The speed of sound in a material in a liquid or a gas is given by,

$$v = (\frac{B}{\rho})^{1/2} \tag{5}$$

where B is the bulk modulus and ρ is the density. In a solid, the sound velocity is given by,

$$v = (\frac{Y}{\rho})^{1/2} \tag{6}$$

where Y is the Young's modulus. Sound travels faster in water or a solid than in air. The speed of sound is temperature dependent and in air the dependence is approximated by(in meters/s),

$$v = 331(\frac{T}{273K})^{1/2} \tag{7}$$

Intensity (I)
    Intensity is equal to power per unit area, and has the units W/m^2. We therefore have,

$$I = \frac{Power}{Area} = \frac{1}{A}\frac{\Delta E}{\Delta t} \tag{8}$$

The limits of human hearing at $1kHz$ are,

$$10^{-12}W/m^2 < I < 1W/m^2 \tag{9}$$

where the lower limit is the faintest sounds that can be heard, while the upper limit is the pain threshold. Acoustic surgery is possible at very high intensities, which requires careful focusing of the acoustic waves. Typical

surgery intensities are of order $I_{surgical} > 10^7 W/m^2$. At these intensities the cell wall breaks and cells fragment. Even medical ultrasound is at intensities of order $100W/m^2$ however the frequency used is well outside the human audible range so it is assumed that no pain is caused by the procedure.

Due to the broad range of sound levels which are relevant to physiology and technology, sound level is usually quoted using a logarithmic scale, the decibel scale (in a similar way earthquake magnitudes are on a logarithmic scale). A sound wave of intensity I in W/m^2 has a decibel value given by,

$$\beta = 10 Log(I/I_0) \tag{10}$$

where $I_0 = 10^{-12} W/m^2$. The threshold of hearing has $\beta = 0$ and the threshold of pain $\beta = 120$.

Often sound is emitted from a point source and the intensity at some distance r from the point source is given by,

$$I(r) = \frac{P}{4\pi r^2} \tag{11}$$

We have already seen this sort of equation when we looked at the power emitted by the sun using the Poisson Boltzmann law, though we did not use the word "intensity". However the use of intensity makes thinking about problems like that somewhat easier.